

EXTENDS *Naturals, Sequences, FiniteSets*

CONSTANTS

*USERIDS,*  
*SERVERS,*  
*METADATAS,*  
*IMAGES,*  
*UUIDS,*  
 Added to  
*CLEANERS*

VARIABLES

*databaseState,*  
*blobStoreState,*  
*serverStates,*  
*cleanerStates,* *CleanerStates[storageCleanerId]*  
  
*operations*

*vars*  $\triangleq$   $\langle$ *databaseState, blobStoreState,*  
*serverStates, operations, cleanerStates* $\rangle$

*cleanerVars*  $\triangleq$   $\langle$ *cleanerStates* $\rangle$

Strong Typing

*UserIdVal*  $\triangleq$  *USERIDS*  $\cup$  {"UNSET"}  
*MetadataVal*  $\triangleq$  *METADATAS*  $\cup$  {"UNSET"}  
*ImageVal*  $\triangleq$  *IMAGES*  $\cup$  {"UNSET"}  
*UUIDVal*  $\triangleq$  *UUIDS*  $\cup$  {"UNSET"} added *UUID* type

*DatabaseRecord*  $\triangleq$  [  
*metadata* : *MetadataVal,*  
*imageId* : *UUIDVal*  
 ]

Describes all possible states a server can be in. Unchanged since last example)

*ServerStateVal*  $\triangleq$   
 [  
   *state* : {  
     current:  
     "waiting", next: *ServerStartWrite* or *ServerStartRead*  
     after: *ServerStartWrite*  
     "started\_write", next: *ServerWriteBlob* or *ServerFailWrite*  
     after: *ServerWriteBlob*

```

    "wrote_blob", next: ServerWriteMetadataAndReturn or ServerFailWrite
  after: ServerStartRead
  "started_read", next: ServerReadMetadata
  after: ServerReadMetadata, ServerReadMetadataAndReturnEmpty
  "read_metadata" next: ServerReadBlobAndReturn
},
userId : UserIdVal,
metadata : MetadataVal,
imageId : UUIDVal, Need to track imageId to perform a lookup
image : ImageVal
]

```

Describes all possible states a server can be in. Unchanged since last example)

```

CleanerStateVal ≜
[
  state : {
    current:
    "waiting", next: CleanerStartGetBlobKeys
  after: waiting
    "got_blob_keys", next: CleanerGetUnusedKeys or CleanerFail
  after: got_blob_keys
    "got_unused_keys", next: CleanerDeleteKeys or CleanerFail
  after: got_unused_keys
  next: CleanerDeleteKeys, CleanerFinished, or waiting
    "deleting_keys"
  },
  blobKeys : SUBSET UUIDS,
  unusedBlobKeys : SUBSET UUIDS
]

```

This is an observability value, and we are still measuring the same thing

No changes are needed

```

OperationValue ≜ [type : { "READ", "WRITE" },
  userId : UserIdVal,
  metadata : MetadataVal,
  image : ImageVal]

```

```

TypeOk ≜
  ∧ databaseState ∈ [USERIDS → DatabaseRecord]
  ∧ blobStoreState ∈ [UUIDS → ImageVal]
  ∧ serverStates ∈ [SERVERS → ServerStateVal]
  Added cleaner states to track status of cleaners
  ∧ cleanerStates ∈ [CLEANERS → CleanerStateVal]
  ∧ operations ∈ Seq(OperationValue)

```



$$\begin{aligned}
& \wedge \text{blobStoreState}[id] = \text{"UNSET"} \mid \\
& \wedge \text{blobStoreState}' = [\text{blobStoreState} \text{ EXCEPT} \\
& \quad \quad \quad \text{![id] = currentState.image}] \\
& \wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT} \\
& \quad \quad \quad \text{![s].state = "wrote_blob",} \\
& \quad \quad \quad \text{![s].imageId = id}] \\
& \wedge \text{UNCHANGED} \langle \text{databaseState}, \text{operations} \rangle \\
& \wedge \text{UNCHANGED} \text{ cleanerVars}
\end{aligned}$$

$$\begin{aligned}
& \text{ServerWriteMetadataAndReturn}(s) \triangleq \\
& \text{LET } \text{currentState} \triangleq \text{serverStates}[s] \\
& \text{IN} \\
& \quad \wedge \text{currentState.state} = \text{"wrote_blob"} \\
& \quad \wedge \text{databaseState}' = [\text{databaseState} \text{ EXCEPT} \\
& \quad \quad \quad \text{![currentState.userId] = [} \\
& \quad \quad \quad \quad \text{metadata} \mapsto \text{currentState.metadata,} \\
& \quad \quad \quad \quad \text{imageId} \mapsto \text{currentState.imageId}}] \\
& \quad \wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT} \\
& \quad \quad \quad \text{![s].state = "waiting",} \\
& \quad \quad \quad \text{![s].userId = "UNSET",} \\
& \quad \quad \quad \text{![s].metadata = "UNSET",} \\
& \quad \quad \quad \text{![s].image = "UNSET",} \\
& \quad \quad \quad \text{![s].imageId = "UNSET"}] \\
& \quad \wedge \text{UNCHANGED} \langle \text{blobStoreState}, \text{operations} \rangle \\
& \quad \wedge \text{UNCHANGED} \text{ cleanerVars}
\end{aligned}$$

$$\begin{aligned}
& \text{ServerFailWrite}(s) \triangleq \\
& \quad \wedge \text{serverStates}[s].\text{state} \in \{ \text{"started_write"}, \text{"wrote_blob"} \} \\
& \quad \wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT} \\
& \quad \quad \quad \text{![s].state = "waiting",} \\
& \quad \quad \quad \text{![s].userId = "UNSET",} \\
& \quad \quad \quad \text{![s].metadata = "UNSET",} \\
& \quad \quad \quad \text{![s].image = "UNSET",} \\
& \quad \quad \quad \text{![s].imageId = "UNSET"}] \\
& \quad \wedge \text{UNCHANGED} \langle \text{databaseState}, \text{blobStoreState}, \text{operations} \rangle \\
& \quad \wedge \text{UNCHANGED} \text{ cleanerVars}
\end{aligned}$$

#### Server Reads

$$\begin{aligned}
& \text{ServerStartRead}(s) \triangleq \\
& \quad \wedge \text{serverStates}[s].\text{state} = \text{"waiting"} \\
& \quad \wedge \exists u \in \text{USERIDS} : \\
& \quad \quad \text{serverStates}' = [\text{serverStates} \text{ EXCEPT} \\
& \quad \quad \quad \text{![s].state = "started_read",}
\end{aligned}$$

```

                                ![s].userId = u

    ∧ UNCHANGED ⟨databaseState, blobStoreState⟩
    ∧ UNCHANGED operations
    ∧ UNCHANGED cleanerVars

    If database record is present
    ServerReadMetadata(s) ≜
      LET currentState ≜ serverStates[s]
      IN
        ∧ currentState.state = "started_read"
        Represents reading the metadata while the database record is set
        ∧ databaseState[currentState.userId].metadata ≠ "UNSET"
        ∧ serverStates' =
          [serverStates EXCEPT
            ![s].state = "read_metadata",
            ![s].metadata = databaseState[currentState.userId].metadata,
            Reads imageId from database
            ![s].imageId = databaseState[currentState.userId].imageId]
        ∧ UNCHANGED ⟨databaseState, blobStoreState⟩
        ∧ UNCHANGED operations
        ∧ UNCHANGED cleanerVars

    If database record is not present
    ServerReadMetadataAndReturnEmpty(s) ≜
      LET currentState ≜ serverStates[s]
      IN
        ∧ currentState.state = "started_read"
        Represents reading the metadata while the database record is unset
        ∧ databaseState[currentState.userId].metadata = "UNSET"
        ∧ serverStates' = [serverStates EXCEPT
          ![s].state = "waiting",
          ![s].userId = "UNSET",
          ![s].metadata = "UNSET",
          ![s].image = "UNSET",
          ![s].imageId = "UNSET"]

        ∧ operations' = Append(operations,
          Returns an empty record
          [
            type ↦ "READ",
            userId ↦ currentState.userId,
            metadata ↦ "UNSET",
            image ↦ "UNSET"
          ])
        ∧ UNCHANGED ⟨databaseState, blobStoreState⟩

```

```

    ∧ UNCHANGED cleanerVars

ServerReadBlobAndReturn(s)  $\triangleq$ 
  LET currentState  $\triangleq$  serverStates[s]
  IN
    ∧ currentState.state = "read_metadata"
    ∧ operations' = Append(operations,
      [
        type  $\mapsto$  "READ",
        userId  $\mapsto$  currentState.userId,
        metadata  $\mapsto$  currentState.metadata,
        image  $\mapsto$  blobStoreState[currentState.imageId]
      ])
    ∧ serverStates' = [serverStates EXCEPT
      ![s].state = "waiting",
      ![s].userId = "UNSET",
      ![s].metadata = "UNSET",
      ![s].image = "UNSET",
      ![s].imageId = "UNSET"]
    ∧ UNCHANGED  $\langle$ databaseState, blobStoreState $\rangle$ 
    ∧ UNCHANGED cleanerVars

```

#### Cleaner States

```

CleanerStartGetBlobKeys(c)  $\triangleq$ 
  LET current  $\triangleq$  cleanerStates[c]
  IN
    Starts only from waiting
    ∧ current.state = "waiting"
    ∧ cleanerStates' = [
      cleanerStates EXCEPT
        ![c].state = "got_blob_keys",
        All keys that are set in blockstore
        ![c].blobKeys = {k ∈ UUIIDS : blobStoreState[k] ≠ "UNSET"}
    ]
    ∧ UNCHANGED  $\langle$ serverStates, databaseState, blobStoreState, operations $\rangle$ 

```

```

CleanerGetUnusedKeys(c)  $\triangleq$ 
  LET current  $\triangleq$  cleanerStates[c]
  IN
    From blob keys, get unused keys from database
    ∧ current.state = "got_blob_keys"
    ∧ cleanerStates' = [
      cleanerStates EXCEPT
        ![c].state = "got_unused_keys",
        ![c].unusedBlobKeys =
          {k ∈ current.blobKeys : Keys in blob keys
           ∨ u ∈ USERIDS : That are not in the database}
    ]

```

$$\begin{aligned}
& \text{databaseState}[u].\text{imageId} \neq k \} \\
& ] \\
& \wedge \text{UNCHANGED} \langle \text{serverStates}, \text{databaseState}, \text{blobStoreState}, \text{operations} \rangle \\
\text{CleanerDeletingKeys}(c) & \triangleq \\
\text{LET } \text{current} & \triangleq \text{cleanerStates}[c] \text{ IN} \\
& \text{When we have unused keys, keep deleting} \\
& \wedge \text{current.state} \in \{ \text{"got\_unused\_keys"}, \text{"deleting\_keys"} \} \\
& \wedge \text{Cardinality}(\text{current.unusedBlobKeys}) \neq 0 \\
& \wedge \exists k \in \text{current.unusedBlobKeys} : \text{pick a key to delete} \\
& \quad \wedge \text{blobStoreState}' = [\text{blobStoreState} \text{ EXCEPT } ![k] = \text{"UNSET"}] \\
& \quad \wedge \text{cleanerStates}' = [ \\
& \quad \quad \text{cleanerStates} \text{ EXCEPT} \\
& \quad \quad \quad \text{remove the key from set} \\
& \quad \quad \quad ![c].\text{unusedBlobKeys} = \text{current.unusedBlobKeys} \setminus \{k\} \\
& \quad ] \\
& \wedge \text{UNCHANGED} \langle \text{serverStates}, \text{databaseState}, \text{operations} \rangle \\
\text{CleanerFinished}(c) & \triangleq \\
\text{LET } \text{current} & \triangleq \text{cleanerStates}[c] \text{ IN} \\
& \text{When we have no more unused keys to delete, finish} \\
& \wedge \text{current.state} = \text{"deleting\_keys"} \\
& \wedge \text{Cardinality}(\text{current.unusedBlobKeys}) = 0 \\
& \wedge \text{cleanerStates}' = [ \\
& \quad \text{cleanerStates} \text{ EXCEPT} \\
& \quad \quad ![c].\text{state} = \text{"waiting"}, \\
& \quad \quad ![c].\text{blobKeys} = \{ \}, \\
& \quad \quad ![c].\text{unusedBlobKeys} = \{ \} \\
& \quad ] \\
& \wedge \text{UNCHANGED} \langle \text{serverStates}, \text{databaseState}, \text{blobStoreState}, \text{operations} \rangle \\
\text{CleanerFail}(c) & \triangleq \\
\text{LET } \text{current} & \triangleq \text{cleanerStates}[c] \text{ IN} \\
& \text{Cleaner can fail from any active state} \\
& \wedge \text{current.state} \in \{ \text{"got\_blob\_keys"}, \text{"got\_unused\_keys"}, \text{"deleting\_keys"} \} \\
& \text{Failure represented by cleaner losing state. Any partial operations stay partially finished.} \\
& \wedge \text{cleanerStates}' = [ \\
& \quad \text{cleanerStates} \text{ EXCEPT} \\
& \quad \quad ![c].\text{state} = \text{"waiting"}, \\
& \quad \quad ![c].\text{blobKeys} = \{ \}, \\
& \quad \quad ![c].\text{unusedBlobKeys} = \{ \} \\
& \quad ] \\
& \wedge \text{UNCHANGED} \langle \text{serverStates}, \text{databaseState}, \text{blobStoreState}, \text{operations} \rangle
\end{aligned}$$

Specification / Next

$Next \triangleq$

For every step, we either trigger a server or cleaner to take a step  
 $\vee \exists s \in SERVERS :$   
     $\vee ServerStartWrite(s)$   
     $\vee ServerWriteBlob(s)$   
     $\vee ServerWriteMetadataAndReturn(s)$   
     $\vee ServerFailWrite(s)$   
     $\vee ServerStartRead(s)$   
     $\vee ServerReadMetadata(s)$   
     $\vee ServerReadMetadataAndReturnEmpty(s)$   
     $\vee ServerReadBlobAndReturn(s)$   
 $\vee \exists c \in CLEANERS :$  all the steps a cleaner can take  
     $\vee CleanerStartGetBlobKeys(c)$   
     $\vee CleanerGetUnusedKeys(c)$   
     $\vee CleanerDeletingKeys(c)$   
     $\vee CleanerFinished(c)$   
     $\vee CleanerFail(c)$

$Spec \triangleq Init \wedge \square[Next]_{vars}$

## Invariants

Note that the success criteria hasn't changed this whole time

$ConsistentReads \triangleq$

If there are no operations, they are consistent  
 $\vee operations = \langle \rangle$   
 $\vee \forall i \in 1 .. Len(operations) :$  For every read operation  
    LET  $readOp \triangleq operations[i]$  IN  
     $\vee \wedge readOp.type = "READ"$   
        There must exist a write operation  
         $\wedge \vee \exists j \in 1 .. i :$   
            LET  $writeOp \triangleq operations[j]$  IN  
             $\wedge writeOp.type = "WRITE"$   
            With the same data  
             $\wedge readOp.userId = writeOp.userId$   
             $\wedge readOp.metadata = writeOp.metadata$   
             $\wedge readOp.image = writeOp.image$   
         $\vee$  Ignore unset reads  
             $\wedge readOp.metadata = "UNSET"$   
             $\wedge readOp.image = "UNSET"$   
     $\vee readOp.type = "WRITE"$  Ignore writes

$NoOrphanFiles \triangleq$



There does not exist a key  
 $\neg \exists k \in UUIDS :$   
 That is in the block store  
 $\wedge blobStoreState[k] \neq \text{"UNSET"}$   
 And not in database  
 $\wedge \forall u \in USERIDS :$   
 $databaseState[u].imageId \neq k$

At some point in the future there will be no orphan files

If it's true ever, it is True

$EventuallyNoOrphanFiles \triangleq \diamond NoOrphanFiles$

Always, at some point in the future, there will be no orphan files

This is how we test eventual consistency. It can't just happen once

It must always happen

$AlwaysEventuallyNoOrphanFiles \triangleq \square EventuallyNoOrphanFiles$

$StopAfter3Operations \triangleq$

$Len(operations) \leq 3$

$StopAfter5Operations \triangleq$

$Len(operations) \leq 5$