

EXTENDS *Naturals, Sequences, FiniteSets*

CONSTANTS

*USERIDS,*  
*SERVERS,*  
*METADATAS,*  
*IMAGES,*  
*UUIDS,*  
*CLEANERS*

VARIABLES

Implementation variables

*databaseState,*  
*blobStoreState,*  
*serverStates,*  
*cleanerStates,*

We just added a time variable here

*time,* Natural number representing the number of hours that have passed

Observability variables

*operations,*  
*usedIds,*

Temporal property variable: used to change state space on failure

*failed*

*helperVars*  $\triangleq$   $\langle$ *time, operations, usedIds, failed* $\rangle$

*vars*  $\triangleq$   $\langle$ *databaseState, blobStoreState,*  
*serverStates, operations, cleanerStates, time, usedIds, failed* $\rangle$

*cleanerVars*  $\triangleq$   $\langle$ *cleanerStates* $\rangle$

Strong Typing

*UserIdVal*  $\triangleq$  *USERIDS*  $\cup$  {"UNSET"}  
*MetadataVal*  $\triangleq$  *METADATAS*  $\cup$  {"UNSET"}  
*ImageVal*  $\triangleq$  *IMAGES*  $\cup$  {"UNSET"}  
*UUIDVal*  $\triangleq$  *UUIDS*  $\cup$  {"UNSET"}

*DatabaseRecord*  $\triangleq$  [  
*metadata* : *MetadataVal,*  
*imageId* : *UUIDVal*  
 ]

$$\text{BlobStoreRecord} \triangleq [$$

$$\quad \text{image} : \text{ImageVal},$$

$$\quad \text{created} : \text{Nat}$$

$$] \cup \{ [$$

$$\quad \text{status} \mapsto \text{"UNSET"},$$

$$\quad \text{image} \mapsto \text{"UNSET"}$$

$$] \}$$

$$\text{ServerStateVal} \triangleq$$

$$[$$

$$\quad \text{state} : \{$$

$$\quad \quad \text{"waiting"},$$

$$\quad \quad \text{"started\_write"},$$

$$\quad \quad \text{"wrote\_blob"},$$

$$\quad \quad \text{"started\_read"},$$

$$\quad \quad \text{"read\_metadata"}$$

$$\quad \},$$

$$\quad \text{userId} : \text{UserIdVal},$$

$$\quad \text{metadata} : \text{MetadataVal},$$

$$\quad \text{imageId} : \text{UUIDVal},$$

$$\quad \text{image} : \text{ImageVal},$$

$$\quad \text{start} : \text{Nat} \quad \text{added to track when a request starts}$$

$$]$$

$$\text{CleanerStateVal} \triangleq$$

$$[$$

$$\quad \text{state} : \{$$

$$\quad \quad \text{"waiting"},$$

$$\quad \quad \text{"got\_blob\_keys"},$$

$$\quad \quad \text{"got\_unused\_keys"},$$

$$\quad \quad \text{"deleting\_keys"}$$

$$\quad \},$$

$$\quad \text{This will be used to introduce a delay}$$

$$\quad \text{unusedKeyTime} : \text{Nat},$$

$$\quad \text{blobKeys} : \text{SUBSET UUIDS},$$

$$\quad \text{unusedBlobKeys} : \text{SUBSET UUIDS}$$

$$]$$

$$\text{OperationValue} \triangleq [ \text{type} : \{ \text{"READ"}, \text{"WRITE"} \},$$

$$\quad \text{userId} : \text{UserIdVal},$$

$$\quad \text{metadata} : \text{MetadataVal},$$

$$\quad \text{image} : \text{ImageVal} ]$$

$$\text{TypeOk} \triangleq$$

$$\quad \wedge \text{databaseState} \in [ \text{USERIDS} \rightarrow \text{DatabaseRecord} ]$$

$$\begin{aligned}
& \wedge \text{blobStoreState} \in [\text{UUIDS} \rightarrow \text{BlobStoreRecord}] \\
& \wedge \text{serverStates} \in [\text{SERVERS} \rightarrow \text{ServerStateVal}] \\
& \wedge \text{cleanerStates} \in [\text{CLEANERS} \rightarrow \text{CleanerStateVal}] \\
& \wedge \text{operations} \in \text{Seq}(\text{OperationValue}) \\
& \wedge \text{time} \in \text{Nat} \\
& \wedge \text{usedIds} \in \text{SUBSET } \text{UUIDS} \\
& \wedge \text{failed} \in \text{Nat}
\end{aligned}$$

$\text{Init} \triangleq$

$$\begin{aligned}
& \wedge \text{databaseState} = \\
& \quad [u \in \text{USERIDS} \mapsto [\text{metadata} \mapsto \text{"UNSET"}, \text{imageId} \mapsto \text{"UNSET"}]] \\
& \wedge \text{blobStoreState} = \\
& \quad [u \in \text{UUIDS} \mapsto [\text{status} \mapsto \text{"UNSET"}, \text{image} \mapsto \text{"UNSET"}]] \\
& \wedge \text{serverStates} = [s \in \text{SERVERS} \mapsto [\text{state} \mapsto \text{"waiting"}, \\
& \quad \text{userId} \mapsto \text{"UNSET"}, \\
& \quad \text{metadata} \mapsto \text{"UNSET"}, \\
& \quad \text{imageId} \mapsto \text{"UNSET"}, \\
& \quad \text{image} \mapsto \text{"UNSET"}, \\
& \quad \text{will be set on start states} \\
& \quad \text{start} \mapsto 0 \\
& \quad ]]] \\
& \wedge \text{cleanerStates} = [c \in \text{CLEANERS} \mapsto [ \\
& \quad \text{state} \mapsto \text{"waiting"}, \\
& \quad \text{blobKeys} \mapsto \{\}, \\
& \quad \text{unusedBlobKeys} \mapsto \{\}, \\
& \quad \text{unusedKeyTime} \mapsto 0 \\
& \quad ]]] \\
& \wedge \text{operations} = \langle \rangle \\
& \wedge \text{time} = 0 \quad \text{Time starts at 0} \\
& \wedge \text{usedIds} = \{\} \\
& \wedge \text{failed} = 0
\end{aligned}$$

State Machine:

$\text{TimePasses} \triangleq$

$$\begin{aligned}
& \wedge \text{time}' = \text{time} + 1 \\
& \wedge \text{UNCHANGED } \langle \text{serverStates}, \text{databaseState}, \\
& \quad \text{blobStoreState}, \text{cleanerStates} \rangle \\
& \wedge \text{UNCHANGED } \langle \text{operations}, \text{usedIds}, \text{failed} \rangle
\end{aligned}$$

Server Restart

$\text{ServerRestart}(s) \triangleq$

$$\begin{aligned}
& \text{LET } \text{currentState} \triangleq \text{serverStates}[s] \text{ IN} \\
& \text{LET } \text{terminationTime} \triangleq (\text{currentState.start} + 1) \text{ IN}
\end{aligned}$$

$\wedge \text{currentState.state} \neq \text{"waiting"}$  Server must be active  
 This is the only state a server can reach if past termination time  
 $\wedge \text{time} \Rightarrow \text{terminationTime}$   
 $\wedge \text{serverStates}' = [\text{serverStates}$  EXCEPT  
     ! $[s].\text{state} = \text{"waiting"}$ ,  
     ! $[s].\text{userId} = \text{"UNSET"}$ ,  
     ! $[s].\text{metadata} = \text{"UNSET"}$ ,  
     ! $[s].\text{image} = \text{"UNSET"}$ ,  
     ! $[s].\text{imageId} = \text{"UNSET"}$ ]  
 $\wedge$  UNCHANGED  $\langle \text{databaseState}, \text{blobStoreState}, \text{cleanerVars} \rangle$   
 $\wedge$  UNCHANGED  $\text{helperVars}$

#### Server Writes

$\text{ServerStartWrite}(s) \triangleq$   
 $\wedge \text{serverStates}[s].\text{state} = \text{"waiting"}$   
 $\wedge \exists u \in \text{USERIDS}, m \in \text{METADATAS}, i \in \text{IMAGES} :$   
      $\wedge \text{serverStates}' = [\text{serverStates}$  EXCEPT  
         ! $[s].\text{state} = \text{"started\_write"}$ ,  
         ! $[s].\text{userId} = u$ ,  
         ! $[s].\text{metadata} = m$ ,  
         ! $[s].\text{image} = i$ ,  
         The time a write request starts  
         ! $[s].\text{start} = \text{time}$   
     ]  
      $\wedge \text{operations}' = \text{Append}(\text{operations},$   
         [  
              $\text{type} \mapsto \text{"WRITE"}$ ,  
              $\text{userId} \mapsto u$ ,  
              $\text{metadata} \mapsto m$ ,  
              $\text{image} \mapsto i$   
         ])  
 $\wedge$  UNCHANGED  $\langle \text{databaseState}, \text{blobStoreState}, \text{cleanerStates} \rangle$   
 $\wedge$  UNCHANGED  $\langle \text{time}, \text{usedIds}, \text{failed} \rangle$

$\text{ServerWriteBlob}(s) \triangleq$   
 LET  $\text{currentState} \triangleq \text{serverStates}[s]$  IN  
 LET  $\text{terminationTime} \triangleq (\text{currentState.start} + 1)$  IN  
 $\wedge \text{time} < \text{terminationTime}$  Can only start this state if server is live  
 $\wedge \text{currentState.state} = \text{"started\_write"}$   
 $\wedge \exists id \in \text{UUIIDS} :$   
      $\wedge id \notin \text{usedIds}$   
      $\wedge \text{blobStoreState}[id] = [\text{status} \mapsto \text{"UNSET"}, \text{image} \mapsto \text{"UNSET"}]$   
      $\wedge \text{blobStoreState}' = [\text{blobStoreState}$  EXCEPT  
         ! $[id] = [$

$$\begin{aligned}
& \text{image} \mapsto \text{currentState.image}, \\
& \text{created} \mapsto \text{time} \\
& \text{]} \\
\wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT} \\
& \quad ![s].\text{state} = \text{"wrote_blob"}, \\
& \quad ![s].\text{imageId} = \text{id}] \\
\wedge \text{usedIds}' = \text{usedIds} \cup \{\text{id}\} \\
\wedge \text{UNCHANGED} \langle \text{databaseState}, \text{cleanerVars} \rangle \\
\wedge \text{UNCHANGED} \langle \text{time}, \text{operations}, \text{failed} \rangle
\end{aligned}$$

$$\begin{aligned}
\text{ServerWriteMetadataAndReturn}(s) \triangleq & \\
\text{LET } \text{currentState} \triangleq & \text{serverStates}[s] \text{ IN} \\
\text{LET } \text{terminationTime} \triangleq & (\text{currentState.start} + 1) \text{ IN} \\
\wedge \text{time} < \text{terminationTime} & \\
\wedge \text{currentState.state} = \text{"wrote_blob"} & \\
\wedge \text{databaseState}' = [\text{databaseState} \text{ EXCEPT} & \\
& \quad ![\text{currentState.userId}] = [ \\
& \quad \quad \text{metadata} \mapsto \text{currentState.metadata}, \\
& \quad \quad \text{imageId} \mapsto \text{currentState.imageId}] \\
\wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT} & \\
& \quad ![s].\text{state} = \text{"waiting"}, \\
& \quad ![s].\text{userId} = \text{"UNSET"}, \\
& \quad ![s].\text{metadata} = \text{"UNSET"}, \\
& \quad ![s].\text{image} = \text{"UNSET"}, \\
& \quad ![s].\text{imageId} = \text{"UNSET"}] \\
\wedge \text{UNCHANGED} \langle \text{blobStoreState}, \text{cleanerVars} \rangle & \\
\wedge \text{UNCHANGED } \text{helperVars} &
\end{aligned}$$

$$\begin{aligned}
\text{ServerFailWrite}(s) \triangleq & \\
\text{LET } \text{currentState} \triangleq & \text{serverStates}[s] \text{ IN} \\
\text{LET } \text{terminationTime} \triangleq & (\text{currentState.start} + 1) \text{ IN} \\
\wedge \text{time} < \text{terminationTime} & \text{ Can only start this state if server is live} \\
\wedge \text{serverStates}[s].\text{state} \in \{ \text{"started_write"}, \text{"wrote_blob"} \} & \\
\wedge \text{serverStates}' = [\text{serverStates} \text{ EXCEPT} & \\
& \quad ![s].\text{state} = \text{"waiting"}, \\
& \quad ![s].\text{userId} = \text{"UNSET"}, \\
& \quad ![s].\text{metadata} = \text{"UNSET"}, \\
& \quad ![s].\text{image} = \text{"UNSET"}, \\
& \quad ![s].\text{imageId} = \text{"UNSET"}] \\
\wedge \text{UNCHANGED} \langle \text{databaseState}, \text{blobStoreState}, \text{cleanerVars} \rangle & \\
\wedge \text{UNCHANGED } \text{helperVars} &
\end{aligned}$$

Server Reads

$$\begin{aligned}
\text{ServerStartRead}(s) &\triangleq \\
&\text{LET } \text{currentState} \triangleq \text{serverStates}[s] \text{ IN} \\
&\wedge \text{serverStates}[s].\text{state} = \text{"waiting"} \\
&\wedge \exists u \in \text{USERIDS} : \\
&\quad \text{serverStates}' = [\text{serverStates EXCEPT} \\
&\quad \quad \quad ! [s].\text{state} = \text{"started\_read"}, \\
&\quad \quad \quad ! [s].\text{userId} = u, \\
&\quad \quad \quad \text{The time a read request starts} \\
&\quad \quad \quad ! [s].\text{start} = \text{time} \\
&\quad \quad \quad ] \\
&\wedge \text{UNCHANGED } \langle \text{databaseState}, \text{blobStoreState}, \text{cleanerVars} \rangle \\
&\wedge \text{UNCHANGED } \text{helperVars}
\end{aligned}$$

$$\begin{aligned}
\text{ServerReadMetadata}(s) &\triangleq \\
&\text{LET } \text{currentState} \triangleq \text{serverStates}[s] \text{ IN} \\
&\text{LET } \text{terminationTime} \triangleq (\text{currentState.start} + 1) \text{ IN} \\
&\wedge \text{time} < \text{terminationTime} \quad \text{Can only start this state if server is live} \\
&\wedge \text{currentState.state} = \text{"started\_read"} \\
&\wedge \text{databaseState}[\text{currentState.userId}].\text{metadata} \neq \text{"UNSET"} \\
&\wedge \text{serverStates}' = \\
&\quad [\text{serverStates EXCEPT} \\
&\quad \quad \quad ! [s].\text{state} = \text{"read\_metadata"}, \\
&\quad \quad \quad ! [s].\text{metadata} = \text{databaseState}[\text{currentState.userId}].\text{metadata}, \\
&\quad \quad \quad ! [s].\text{imageId} = \text{databaseState}[\text{currentState.userId}].\text{imageId}] \\
&\wedge \text{UNCHANGED } \langle \text{databaseState}, \text{blobStoreState}, \text{cleanerVars} \rangle \\
&\wedge \text{UNCHANGED } \text{helperVars}
\end{aligned}$$

$$\begin{aligned}
\text{ServerReadMetadataAndReturnEmpty}(s) &\triangleq \\
&\text{LET } \text{currentState} \triangleq \text{serverStates}[s] \text{ IN} \\
&\text{LET } \text{terminationTime} \triangleq (\text{currentState.start} + 1) \text{ IN} \\
&\wedge \text{time} < \text{terminationTime} \quad \text{Can only start this state if server is live} \\
&\wedge \text{currentState.state} = \text{"started\_read"} \\
&\wedge \text{databaseState}[\text{currentState.userId}].\text{metadata} = \text{"UNSET"} \\
&\wedge \text{serverStates}' = [\text{serverStates EXCEPT} \\
&\quad \quad \quad ! [s].\text{state} = \text{"waiting"}, \\
&\quad \quad \quad ! [s].\text{userId} = \text{"UNSET"}, \\
&\quad \quad \quad ! [s].\text{metadata} = \text{"UNSET"}, \\
&\quad \quad \quad ! [s].\text{image} = \text{"UNSET"}, \\
&\quad \quad \quad ! [s].\text{imageId} = \text{"UNSET"}] \\
&\wedge \text{operations}' = \text{Append}(\text{operations}, \\
&\quad [ \\
&\quad \quad \text{type} \mapsto \text{"READ"}, \\
&\quad \quad \text{userId} \mapsto \text{currentState.userId}, \\
&\quad ]
\end{aligned}$$

```

        metadata ↦ "UNSET",
        image ↦ "UNSET"
    ])
    ∧ UNCHANGED ⟨databaseState, blobStoreState, cleanerVars⟩
    ∧ UNCHANGED ⟨usedIds, time, failed⟩

ServerReadBlobAndReturn(s) ≜
    LET currentState ≜ serverStates[s]IN
    LET terminationTime ≜ (currentState.start + 1)IN
    ∧ time < terminationTime Can only start this state if server is live
    ∧ currentState.state = "read_metadata"
    ∧ operations' = Append(operations,
        [
            type ↦ "READ",
            userId ↦ currentState.userId,
            metadata ↦ currentState.metadata,
            image ↦ blobStoreState[currentState.imageId].image
        ])
    ∧ serverStates' = [serverStates EXCEPT
        ![s].state = "waiting",
        ![s].userId = "UNSET",
        ![s].metadata = "UNSET",
        ![s].image = "UNSET",
        ![s].imageId = "UNSET"]
    ∧ UNCHANGED ⟨databaseState, blobStoreState, cleanerVars⟩
    ∧ UNCHANGED ⟨usedIds, time, failed⟩

```

#### Cleaner States

This is the main change in the logic.

```

CleanerStartGetBlobKeys(c) ≜
    LET current ≜ cleanerStates[c]IN
    ∧ current.state = "waiting"
    ∧ cleanerStates' = [
        cleanerStates EXCEPT
        ![c].state = "got_blob_keys",
        All keys in blockstore
        ![c].blobKeys = {
            k ∈ UUIDS :
                LET earliestDeletionTime ≜ blobStoreState[k].created + 2IN
                That are not unset
                ∧ blobStoreState[k] ≠ [
                    status ↦ "UNSET",
                    image ↦ "UNSET"]
                It must be created 2 or more hours ago
        }
    ]

```

$$\begin{aligned}
& \wedge \text{earliestDeletionTime} \leq \text{time} \\
& \} \\
& ] \\
& \wedge \text{UNCHANGED} \langle \text{serverStates}, \text{databaseState}, \\
& \quad \text{blobStoreState} \rangle \\
& \wedge \text{UNCHANGED} \text{ helperVars} \\
\text{CleanerGetUnusedKeys}(c) \triangleq & \\
\text{LET } \text{current} \triangleq \text{cleanerStates}[c] \text{ IN} & \\
\wedge \text{current.state} = \text{"got_blob_keys"} & \\
\wedge \text{cleanerStates}' = [ & \\
\quad \text{cleanerStates EXCEPT} & \\
\quad \quad ![c].\text{state} = \text{"got_unused_keys"}, & \\
\quad \quad ![c].\text{unusedBlobKeys} = & \\
\quad \quad \quad \{k \in \text{current.blobKeys} : & \\
\quad \quad \quad \quad \forall u \in \text{USERIDS} : & \\
\quad \quad \quad \quad \quad \text{databaseState}[u].\text{imageId} \neq k\}, & \\
\quad \quad \quad \text{Mark the time the unused keys were retrieved} & \\
\quad \quad \quad ![c].\text{unusedKeyTime} = \text{time} & \\
& ] \\
& \wedge \text{UNCHANGED} \langle \text{serverStates}, \text{databaseState}, \\
& \quad \text{blobStoreState} \rangle \\
& \wedge \text{UNCHANGED} \text{ helperVars} \\
\text{CleanerDeletingKeys}(c) \triangleq & \\
\text{LET } \text{current} \triangleq \text{cleanerStates}[c] \text{ IN} & \\
\text{Keys get deleted a minimum 1 hour after they are valid. Giving reads} & \\
\text{a the time to die.} & \\
\text{LET } \text{earliestDeleteTime} \triangleq \text{current.unusedKeyTime} + 1 \text{ IN} & \\
\wedge \text{time} \geq \text{earliestDeleteTime} & \\
\wedge \text{current.state} \in \{\text{"got_unused_keys"}, \text{"deleting_keys"}\} & \\
\wedge \text{Cardinality}(\text{current.unusedBlobKeys}) \neq 0 & \\
\wedge \exists k \in \text{current.unusedBlobKeys} : \text{pick a key to delete} & \\
\wedge \text{blobStoreState}' = & \\
\quad [\text{blobStoreState EXCEPT} & \\
\quad \quad ![k] = [\text{status} \mapsto \text{"UNSET"}, \text{image} \mapsto \text{"UNSET"}]] & \\
\wedge \text{cleanerStates}' = [ & \\
\quad \text{cleanerStates EXCEPT} & \\
\quad \quad ![c].\text{unusedBlobKeys} = \text{current.unusedBlobKeys} \setminus \{k\} & \\
& ] \\
& \wedge \text{UNCHANGED} \langle \text{serverStates}, \text{databaseState} \rangle \\
& \wedge \text{UNCHANGED} \text{ helperVars} \\
\text{CleanerFinished}(c) \triangleq & \\
\text{LET } \text{current} \triangleq \text{cleanerStates}[c] \text{ IN} & \\
\wedge \text{current.state} = \text{"deleting_keys"} &
\end{aligned}$$



$$\begin{aligned}
& \wedge \text{Cardinality}(\text{current.unusedBlobKeys}) = 0 \\
& \wedge \text{cleanerStates}' = [ \\
& \quad \text{cleanerStates EXCEPT} \\
& \quad \quad ![c].\text{state} = \text{"waiting"}, \\
& \quad \quad ![c].\text{blobKeys} = \{\}, \\
& \quad \quad ![c].\text{unusedBlobKeys} = \{\} \\
& \quad ] \\
& \wedge \text{UNCHANGED} \langle \text{serverStates}, \text{databaseState}, \\
& \quad \quad \text{blobStoreState} \rangle \\
& \wedge \text{UNCHANGED} \text{ helperVars} \\
\text{CleanerFail}(c) & \triangleq \\
\text{LET } \text{current} & \triangleq \text{cleanerStates}[c] \text{ IN} \\
& \wedge \text{current.state} \in \{\text{"got_blob_keys"}, \text{"got_unused_keys"}, \text{"deleting_keys"}\} \\
& \wedge \text{cleanerStates}' = [ \\
& \quad \text{cleanerStates EXCEPT} \\
& \quad \quad ![c].\text{state} = \text{"waiting"}, \\
& \quad \quad ![c].\text{blobKeys} = \{\}, \\
& \quad \quad ![c].\text{unusedBlobKeys} = \{\} \\
& \quad ] \\
& \text{change state space on failed} \\
& \wedge \text{failed}' = \text{failed} + 1 \\
& \wedge \text{UNCHANGED} \langle \text{serverStates}, \text{databaseState}, \\
& \quad \quad \text{blobStoreState} \rangle \\
& \wedge \text{UNCHANGED} \langle \text{time}, \text{operations}, \text{usedIds} \rangle
\end{aligned}$$

Specification / Next

$$\begin{aligned}
\text{CleanerSteps} & \triangleq \\
& \exists c \in \text{CLEANERS} : \\
& \quad \vee \text{CleanerStartGetBlobKeys}(c) \\
& \quad \vee \text{CleanerGetUnusedKeys}(c) \\
& \quad \vee \text{CleanerDeletingKeys}(c) \\
& \quad \vee \text{CleanerFinished}(c) \\
& \quad \vee \text{CleanerFail}(c) \\
\text{Next} & \triangleq \\
& \text{Time can pass now} \\
& \vee \text{TimePasses} \\
& \vee \exists s \in \text{SERVERS} : \\
& \quad \vee \text{ServerStartWrite}(s) \\
& \quad \vee \text{ServerWriteBlob}(s) \\
& \quad \vee \text{ServerWriteMetadataAndReturn}(s) \\
& \quad \vee \text{ServerFailWrite}(s) \\
& \quad \vee \text{ServerStartRead}(s) \\
& \quad \vee \text{ServerReadMetadata}(s)
\end{aligned}$$

$\vee \text{ServerReadMetadataAndReturnEmpty}(s)$   
 $\vee \text{ServerReadBlobAndReturn}(s)$   
 $\vee \text{CleanerSteps}$

$\text{Spec} \triangleq \wedge \text{Init}$   
 $\wedge \square[\text{Next}]_{\text{vars}}$   
 $\wedge \text{WF}_{\text{vars}}(\text{CleanerSteps})$  The cleaner will always get to run

### Invariants

Note that the success criteria hasn't changed this whole time

$\text{ConsistentReads} \triangleq$   
If there are no operations, they are consistent  
 $\vee \text{operations} = \langle \rangle$   
 $\vee \forall i \in 1 \dots \text{Len}(\text{operations}) :$  For every read operation  
 $\text{LET } \text{readOp} \triangleq \text{operations}[i] \text{ IN}$   
 $\vee \wedge \text{readOp.type} = \text{"READ"}$   
There must exist a write operation  
 $\wedge \vee \exists j \in 1 \dots i :$   
 $\text{LET } \text{writeOp} \triangleq \text{operations}[j] \text{ IN}$   
 $\wedge \text{writeOp.type} = \text{"WRITE"}$   
With the same data  
 $\wedge \text{readOp.userId} = \text{writeOp.userId}$   
 $\wedge \text{readOp.metadata} = \text{writeOp.metadata}$   
 $\wedge \text{readOp.image} = \text{writeOp.image}$   
 $\vee$  Ignore unset reads  
 $\wedge \text{readOp.metadata} = \text{"UNSET"}$   
 $\wedge \text{readOp.image} = \text{"UNSET"}$   
 $\vee \text{readOp.type} = \text{"WRITE"}$  Ignore writes

$\text{NoOrphanFiles} \triangleq$   
There does not exist a key  
 $\neg \exists k \in \text{UUIIDS} :$   
That is in the block store  
 $\wedge \text{blobStoreState}[k] \neq [\text{status} \mapsto \text{"UNSET"}, \text{image} \mapsto \text{"UNSET"}]$   
And not in database  
 $\wedge \forall u \in \text{USERIDS} :$   
 $\text{databaseState}[u].\text{imageId} \neq k$

This is used for model checker configuration so that simulation doesn't go on forever

$\text{EventuallyNoOrphanFiles} \triangleq \square \diamond \text{NoOrphanFiles}$

$\text{StopAfter3Operations} \triangleq$

$$\wedge Len(operations) \leq 3$$
$$\wedge time \leq 3$$
$$StopAfter5Operations \triangleq$$
$$Len(operations) \leq 5$$

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